This question paper contains 7 printed pages]

Roll No. | | | | | | | | | | |

S. No. of Question Paper : **6070**

Unique Paper Code : **234301** **D**

Name of the Paper : **Design and Analysis of Algorithms (CSHT-305)**

Name of the Course : **B.Sc. (Hons.) Computer Science**

Semester : **III**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Question No. **1** of **35** marks is compulsory.

Attempt any *four* questions from Q. No. **2** to Q. No. **7**.

1. (*a*) Prove that the average running time of Quicksort is O(n lg n) where n is the number of elements. 5

(*b*) Determine an LCS of <1, 0, 0, 1, 0, 1, 0, 1> and <0, 1, 0, 1, 1, 0, 1, 1, 0>. 5

(*c*) Beginning with an empty red-black tree insert in succession the following keys : 5

10 20 30 80 50 70 75

From the above tree delete 20.

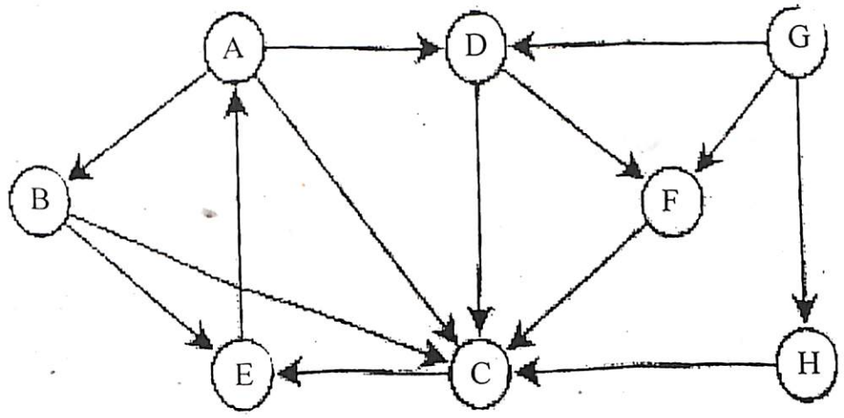
P.T.O.

(*d*) Consider the following recursive function :

```
TOH(in n, char from, char to, char aux)
{
    if (n==1)
        Cout<<"Move disk 1 from "<<from<<" to "<<to;
    else
    {
        TOH (n-1, from,aux,to);
        cout<<"Move disk"<<n<<" from"<<from<<"to"<<to;
        TOH(n-1,aux,to,from);
    }
}
```

Let $T(n)$ represents the total number of moves required to move n disks from one ower to another. Give a recurrence relation for T(n). Find T(n) in terms of ›ig Oh.

(*e*) Apply DFS algorithm on the following graph. Assume that adjacency lists are in alphabetical order. Also classify the different edges as tree, forward, back and cross edge. 5



(*f*) Is it always true that an array that is already sorted is a best-case input for sorting algorithms ? Give an argument or a counterexample. 3

(*g*) Suppose the elements in an array are (starting at index 1) 25, 19, 15, 5, 12, 4, 13, 3, 7, 10. Does this array represent a heap ? Justify your answer. 2

(*h*) Suppose we perform a sequence of n operations on a data structure in which the i^{th} operation costs i if i is an exact power of 2 and 1 otherwise. Use aggregate analysis to determine the amortized cost per operation. 5

ppose that you are given n red and n blue water jugs, all of different shapes and
zes. All red jugs hold different amounts of water, as do the blue ones. Moreover, for
ery red jug, there is a blue jug that holds the same amount of water, and vice versa.
is your task to find a grouping of the jugs into pairs of red and blue jugs that hold
e same amount of water.

do so, you may perform the following operation : pick a pair of jugs in which
e is red and one is blue, fill the red jug with water, and then pour the water
o the blue jug. This operation will tell you whether the red or the blue jug can hold
re water, or if they are of the same volume. Assume that such a comparison takes
e time unit. Your goal is to give an algorithm that uses $O(n^2)$ comparisons to group
jugs into pairs. Remember that you may not directly compare two red jugs or two
e jugs. 5

d the average/expected time taken by bucket sort to sort an array containing
ements. 5

pose we are to find the k smallest elements in a list of n elements, and we are *not*
ested in their relative order. Give a linear time algorithm for the same. 5

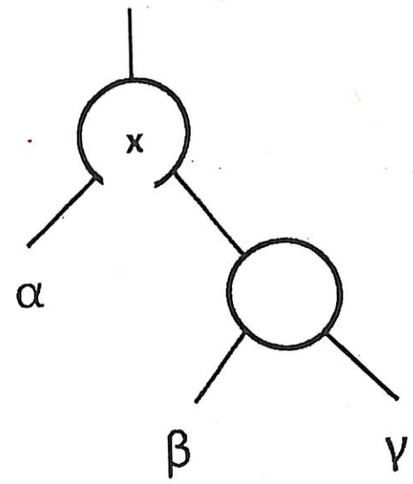
pose that CONNECTED-COMPONENTS is run on the undirected graph
(V, E), where V = {*a*, *b*, *c*, *d*, *e*, *f*, *g*, *h*, *i*, *j*, *k*} and the edges of E are
essed in the following order : (*d*, *i*), (*f*, *k*), (*g*, *i*), (*b*, *g*), (*a*, *h*), (*i*, *j*), (*d*, *k*),
, (*d*, *f*), (*g*, *j*), (*a*, *e*), (*i*, *d*). List the vertices in each connected component
each iteration. 5

(*a*) Suppose that we have numbers between 1 and 50 in a red-black tree and want to search for the number 19. Which of the following sequences could not be the sequence of nodes examined ?

(*i*) 26, 28, 15, 17, 19

(*ii*) 35, 12, 28, 17, 19. 2

(*b*) Let *a*, *b* and *c* be arbitrary nodes in subtrees α, β and γ respectively, in the following figure. How do the depths of *a*, *b* and *c* change when a left rotation is performed on node x in the figure ? 3



(*c*) What is the largest possible number of internal nodes in a red-black tree with black-height k ? What is the smallest possible number of nodes ? 3

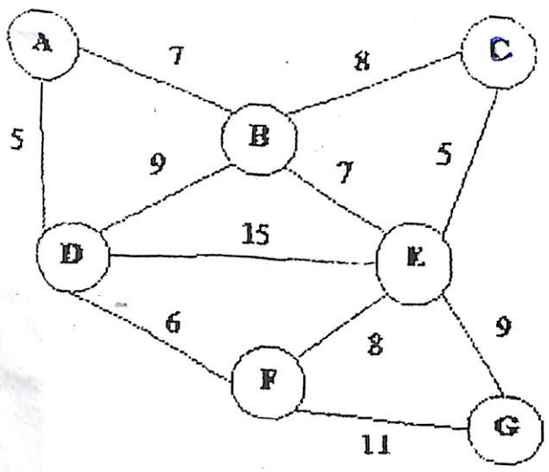
(*d*) What is an order-statistic tree ? Explain and give an example. 2

P.T.O.

What is memorization ? Show how this helps in increasing the efficiency of a recursive program. 5

Not just any greedy approach to the activity-selection problem produces a maximum-size set of mutually compatible activities. Give an example to show that the approach of selecting the activity of least duration from those that are compatible with previously selected activities does not work. 5

Find the minimum spanning tree for the following graph using Kruskal's algorithm. 5



A bipartite graph is a graph whose vertices may be partitioned into two subsets such that there is no edge between any two vertices in the same subset. Write an algorithm to find whether a given directed graph is bipartite or not. 5

7. (*a*) Find the failure indexes for the following pattern :

AABAACAABABA

Assume array indexes are starting from 1. 5

(*b*) Is Prim's minimum spanning tree algorithm a greedy algorithm ? Justify your answer. Also give running time of the algorithm. 5